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Exploring Cognitive Strategy Instruction (CSI), Schema-Based Instruction (SBI) and Strategic Content Learning (SCL) with Students with Learning and Developmental Disabilities in Higher-Order Mathematics:

Two Interrelated Action Research Projects

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Abstract

Students with learning disabilities and developmental disabilities often experience challenges in conducting higher-level mathematic activities, such as solving word problems. These difficulties are usually attributable to deficits in metacognitive processes. Fortunately, there is a promising class of interventions (evidence-based practices) in this area that are variably known as cognitive strategy instruction (CSI), schema-based instruction (SBI), strategic content learning (SCL), etc. The purpose of this study was to explore the use of these strategies with students with learning challenges in higher-order mathematical problems. Specifically, we applied these strategies in two local classrooms: a high school math classroom specifically for students with learning disabilities and a mainstream elementary school classroom that contained some students with special needs. In the high school classroom we also included some concrete-representational-abstract (CRA) instructional sequencing. We found these strategies to be very successful in both classrooms.

Introduction

Students with learning difficulties, such as learning disabilities and developmental disabilities, may experience challenges in recalling basic math facts and comprehending language. These challenges may contribute to ineffective deployment of metacognitive strategies. Therefore, students with learning difficulties may be at risk of under-performing in activities using higher-level mathematics concepts, such as solving word problems or algebra.

Recently, researchers and educational advocates, such as Montague & Jitendra (2007), Foegen (2007) and Schoenfeld (2002), have argued that algebra and what may be broadly labeled as higher-order mathematical concepts need to be conceptualized more as a tool domain, and less as a content area. Competency in algebra and higher-order mathematics permits students a wide range of post-secondary training opportunities that may otherwise be thwarted. Following this logic, effective and inclusive mathematics instruction for students with learning difficulties is a matter of equity – indeed, a matter of rights. The notion of accessible and/or effective mathematics instruction as an issue of social justice is a very different way of thinking about the responsibility of teaching math, particularly complex concepts.

We took up this general challenge as the focus of our two interrelated action research projects.

Best Practices in Mathematics Instruction for Students with Learning Difficulties

Probably due to the significance of higher-order mathematic proficiency, several researchers have thought about best practices, evidence-based practices, in the context of students with learning difficulties. For example, Maccini and Gagnon (2000) argue that "effective instruction techniques, manipulatives, and real-life application" (p. 2) are three significant dimensions of best practice in mathematics. More recently, Ketterlin-Geller, Chard and Fien (2008, p. 35) extensively surveyed relevant literature in the area; they suggest that "six instructional strategies… as beneficial for students with disabilities," the top three of which are central to our investigation: "visual and graphic depictions, systematic and explicit instruction, [and] student think alouds." Ketterlin-Geller et al. (2008) also claim that "other instructional strategies have been documented to be effective in supporting mathematics learning for students struggling with learning mathematics… Researchers have demonstrated that a graduated instructional sequence that proceeds from concrete to representational" (p. 35) is very useful.

To greater and lesser degrees, cognitive and metacognitive instruction in mathematics for those who struggle typically includes these aforementioned dimensions. Also referred to as academic self-regulation, metacognition is recognized as a critical aspect of becoming an effective learner, and involves such things as ongoing analyses of the task demands, strategy implementation and self-monitoring. Although not often conceptualized as a class of interventions within the academic literature, we saw three approaches as convincingly related and constituting the best practices variables of visual representation, systematic instruction and ongoing strategic assessment through think alouds. These three approaches were cognitive strategy instruction (CSI), schema-based instruction

(SBI), and strategic content learning (SCL). Each of these approaches was crucial to our investigation.

Overview of Cognitive Strategy Instruction (CSI), Schema-Based Instruction (SBI) and Strategic Content Learning (SCL)

Each of these instructional approaches comprises a number of strategies or sub-components, usually presented in a sequence. CSI/SBI/SCL may be thought of as amalgams of various instructional strategies, with the way that each one is articulated depending on the teacher, the students and the context. It is not our intention to compare the efficacies of these approaches or overly subscribe to their operational/procedural definitions. For us, the value and relevance of CSI/SBI/SCL does not lie in being able to cleanly stake strategies as integral and fundamental only to CSI or only to SBI or only to SCL, etc.; rather, we see the strength of each approach in how it can shape ongoing instructional practices in the classroom. In our two interrelated studies, our work in the two classrooms "borrowed" strategies from all three approaches, as classroom instruction within the everyday realities of the school often does. Having said that, however, it may be useful to highlight some of the theoretical underpinnings most closely associated with each of these three instructional approaches.

We would argue that CSI, SBI and SCL are each premised upon socio-cultural and constructivist assumptions about teaching and learning and, to a lesser extent, have theoretical underpinnings in cognitive psychology (especially CSI and SBI). As such, they have several key features in common. They emphasize contextualized, meaningful content, and the social aspect of learning is significant. However, within CSI and SBI there is typically some explicit teaching of conceptual/procedural knowledge as well as "think aloud" teacher-mediated protocols, the use of schema and/or other concrete/visual representational strategies, and scaffolded practice with teacher and/or peers (Owen & Fuchs, 2002), which is usually followed by independent practice. On the other hand, instructional strategies within SCL appear to be less explicit, although definitely scaffolded; the role of the teacher/tutor is not so much to model and demonstrate particular strategies, but to support learners in discerning, constructing and creating their own (Butler, 2002/2003; Butler, Beckingham, & Novak Lauscher, 2005). As a result, it may be helpful to conceptualize CSI and SBI as slightly more informed by cognitive theoretical principles than SCL, although there is most certainly socio-cultural understandings inherent with each. It is also noteworthy that CSI and SCL are general approaches that may be used in a variety of content areas, whereas schema-based instruction, as will be indicated though our literature review seems to be located mainly within mathematical word problems.

A detailed description of each of these approaches is given below.

Cognitive Strategy Instruction

CSI, which is sometimes referred to as general strategy instruction (GSI), may be described as a way of supporting students to direct their actions to meet learning goals. Simply put, "a strategy is a tool, plan, or method used for accomplishing a task" (Council for Exceptional Children [CEC], 2002, p. 2); it tends to concentrate and enhance effort. A cognitive strategy is "a strategy or group of strategies or procedures that the learner uses to perform academic tasks" (CEC, 2002, p. 2). Strategy instruction typically includes "teaching students about strategies, teaching them how and when to use strategies, helping students identify personally effective strategies, and encouraging them to make strategic behaviors part of their learning schema" (CEC, 2002, p. 3). Ultimately, the hope of CSI is to enable all students to become more strategic, flexible and independent in their learning processes.

An effective learner may be described as one who has a variety of learning strategies and uses them to meet cognitive challenges. Many types of strategies may be used, such as visualization, verbalization, making associations, chunking, questioning, scanning, underlining, accessing cues, mnemonics, etc. The learner enacts agency in invoking different strategies at key moments throughout the learning endeavor. This agency is a conscious, knowing and, ideally, easily controllable process. According to the Council for Exceptional Children (2002), the basic steps in teaching strategy use are:

- *Describe the strategy*. Students obtain an understanding of the strategy and its purpose; they know why it is important, when it can be used, and how to use it.
- *Model its use*. The teacher models the strategy, explaining to the students how to carry it out.
- *Provide ample assisted practice time*. The teacher monitors, provides cues, and gives feedback. Practice results in automaticity so the student doesn't have to "think" about using the strategy.
- *Promote student self-monitoring and evaluation of personal strategy use*. Students will likely use the strategy if they see how it works for them; it will become part of their learning schema.
- *Encourage continued use and generalization of the strategy*. Students are encouraged to try the strategy in other learning situations (p. 5).

Cognitive Strategy Instruction in Mathematics Word Problems

Cognitive strategy instruction "often leads to improved student performance…[in] computation and problem solving…, reading accuracy and fluency…, and reading comprehension" (CEC, 2002, p. 4). It may not be surprising then that CSI is often employed (and researched) in the context of higher-order mathematics and students with learning difficulties (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Hutchinson, 1993; Jitendra, DiPipi & Perron-Jones, 2002; Jitendra, 2002; Ketterlin-Geller et al., 2008; Morin & Miller, 1998; Owen & Fuchs, 2002; Ping Xin, Jitendra & Deatline-Buchma, 2005; Uberti, Mastropieri & Scruggs, 2004). Students with learning disabilities and/or developmental disabilities generally have difficulty in representing mathematical problems, deriving goals for solving such problems, choosing among appropriate strategies for problem solving, and engaging in self-monitoring processes.

Hutchinson (1993) may well have been one of the first researchers to teach adolescents struggling with algebra and word problems. Recognizing that students with learning disabilities have challenges in recognizing the mathematical structures within word problems and tend to respond to the surface or narrative (storied) aspects of word problems, she focused CSI efforts into supporting students to construct meaningful (mathematically undergirded) schema. Hutchinson taught students to break down the process into steps. More particularly, they were engaged in word problem representation by drawing a picture or constructing a chart or table and then went on to solve the problem. In her study, all students were proficient at basic math operations and one-operation word problems, but not at more complex word problems. Hutchinson (1993) provided general scripts for each type of math problem that included information about how to first represent algebra problems and then solve them. For approximately four months, she conducted individual cognitive strategy instruction sessions with 12 students struggling with algebra; she began by demonstrating and describing strategies, then modeling them through think alouds, moving to guided practice (scaffolded practice with feedback) and eventually to independent practice (1993, p. 41). Hutchinson's study appears to be a landmark one; her data are very thorough and comprehensive, and her results are convincing indeed! Most researchers have adopted her general approach and tend to instruct problem representation, problem solving, and generating the final answer in separate phases (Jitendra, DiPipi & Perron-Jones, 2002; Owen & Fuchs, 2002; Ping Xin, et al., 2005). Of note to our study, Hutchinson tape-recorded the think alouds and provided two cue cards and a worksheet to support students' metacognitive activities, referring to these as "self-questions for representing algebra word problems, self-questions for solving algebra word problems" and a "structured goal worksheet," respectively (1993, pp. 39-40).

Schema-Based Instruction (SBI) in Mathematics Word Problems

A few years after Hutchinson's 1993 study, Sandra Marshall published a very interesting book entitled Schemas in Problem-Solving (1995). Marshall surveyed a large number of math textbooks used in schools, paying attention to the word problems. From this broad overview she developed five fairly "standard" schemas:

> *Chapter 3 presents specific schemas of arithmetic story problems…The situations that underlie the schemas were identified through an examination of existing texts and other materials currently in use in today's classrooms…This method of identifying schemas depends upon meaningful ways of characterizing the problems that occur in the existing environment and reflects the underlying assumption that this environment, or at least problems within it are likely to remain much the same over time (Marshall, 1995, p. 2).*

Several researchers in education have taken up Marshall's constructed schema, (Ping Xin, et al., 2005; Jitendra & Beck, 1999, Jitendra, 2002) in order to support students struggling with higher-order mathematics. It appears that the kinds of challenges identified by Hutchinson in 1993 remain salient today, particularly the tendency for students to pay attention to the storied aspects of word problems and not the mathematical structures:

> *Ineffective instructional strategies may explain the poor problem solving performance of students with learning disabilities. One commonly used instructional approach is the "key word" strategy. For example, students learn that altogether indicates the use of the addition operation, whereas left indicates subtraction. Similarly, the word times calls for multiplication, and among indicates the need to divide. However, the outcome is that students react to the cue word at a surface level of analysis and fail to perform a deep-structure analysis of the interrelationships among the word and the context" (Ping Xin, et al., 2005, p. 181).*

Not surprisingly, advocates of SBI suggest that Marshall's schemas are the way to teach to the deeper mathematical structures. It would seem that, at least in the context of mathematics higher-order instruction, schema-based instruction often refers to Marshall's five schemas, i.e., students are taught to invoke these schemas and not to construct their own as Hutchinson proposed.

> *Overall, both groups were taught to follow the four-step general problem solving procedure of reading to understand, representing the*

> *problem, and planning, solving, and checking. However, the fundamental differences between the two conditions involved the second and third steps, with regard to how to plan and solve the problem. Specifically, the SBI group was taught to identify the problem structure and use a schema diagram to represent and solve the problem, whereas the GSI [general strategy instruction] group learned to draw semiconcrete pictures to represent information in the problem and facilitate problem solving. (Ping Xin, et al., 2005, p. 183).*

Ping Xin, et al. (2005) found SBI to be superior to general strategy instruction, which is very close to CSI; however, as educational researchers and classroom teachers, we certainly saw the benefits of both CSI and SBI. In our view there are more similarities than there are differences between them. SBI seems a little more structured in that the schemas are already created, and such structure is useful for students with learning difficulties. CSI, used in the context of math word problem solving, allows students a little more flexibility in creating their own visual representations. Our two interrelated projects borrowed aspects from each approach, as well as strategic content learning, which is the topic of the next section.

Strategic Content Learning

Strategic Content Learning (SCL) is also, as its name implies, a strategy training approach, but "in contrast to other strategy training models, task specific strategies appropriate for individual students are not determined in advance. Instead, students and instructors establish an understanding of the task, define task goals, and then, using task goals as a foundation for decision making, select, adapt, or invent task-specific strategies." (Kamann & Butler, 1996, p. 5). To state it another way:

> *SCL instruction is geared, not toward teaching students specific, predefined strategies, but instead, towards supporting students to generate personalized strategies for themselves as they self-regulate their engagement in tasks. At the same time, SCL is not like discovery learning, during which students are placed in situations where they might develop strategies on their own. Rather, drawing on their knowledge of the variety of ways in which tasks can be accomplished, SCL instructors provide guidance to students to develop strategies as they work through meaningful tasks. During SCL instruction, students are ultimately responsible for making decisions and judgments regarding their task performance, regarding what goals to set, what strategies to use, how effectively they are progressing towards goals, and how to adapt strategies accordingly (Butler, 1996, p. 5).*

An obvious question is what does teacher guidance look like in SCL, if it is not modelling strategies through think aloud protocols, etc. as in SBI and CSI? Simply put, SCL may be thought of as a gentle, subtle kind of contextual prompting. Kamann & Butler (1996) present several categories of strategies: the first is direct and guiding questions and comments, and the second is supportive and organizational questions and comments. Direct questions and comments, such as explaining and evaluating, are to be used sparingly in favour of guiding strategies, such as guiding, interpreting or linking content and/or procedures. Supportive and organizational questions and comments, so-named as open, paraphrased and supportive, are also to be employed. SCL tends to occur over a series of sessions. In the beginning guiding strategies are suggested, and towards the end, as students become more independent, supportive and organizational strategies are recommended (see Kamann & Butler 1996, pp. 9-11). Over several studies, mostly completed by Butler and a variety of colleagues (Butler, Beckingham & Novak Lauscher, 2005; Butler, 2003/2002/1996; and Kamann & Butler, 1996), SCL has shown to be an evidence-based practice.

We now turn to a brief look at instructional sequencing that we used in our second action research project.

Graduated Instructional Sequencing: Moving from Concrete to Representational to Abstract (CRA)

Again, Ketterlin-Geller et al. (2008) and many others espouse a thoughtful, planned movement from concrete to representational to abstract thinking (CRA) as part of best practice in higher-order mathematics (see also Maccini & Gagnon, 2000; Morin & Miller, 1998). In the concrete phase, students represent the problem with concrete objects or manipulatives; in the representational (sometimes referred to as the semi-concrete) phase, students draw or use pictorial representations of the quantities; and finally, during the abstract phase, students use numeric representations instead of pictorial displays. CRA is often integrated with strategy instruction, as we shall see later in our second action research project.

Research clearly demonstrates that CRA sequencing is useful for teaching elementary mathematics, but significantly here, also for teaching higher-order math concepts with middle years and secondary students. Despite this evidence, many secondary and middle years' teachers do not regularly employ the CRA teaching strategy. Hardy (2005) gives some possible reasons:

- Teacher mistrust of the usefulness or efficiency of manipulative objects for *higher-level algebra*.
- Classroom limitations involving rigid schedules, movement of students and teachers, and the organization and supply of manipulatives.
- Dominance of textbook lessons.
- *Confidence* of teachers in their mathematics knowledge compared to confidence in the use of manipulatives.
- Frequency of use, with some research indicating that secondary teachers use manipulatives *once a month* while primary teachers use them daily.

Purpose

The overarching purpose of our research was to explore the application of CSI/SBI/SCL (and CRA in the second project) in the everyday classroom. Given the reported efficacies of these approaches, it is quite surprising that they appear not to be routinely employed:

> Currently, there are little data available to determine how many teachers teach strategic learning skills, how many are even aware of their existence, or if they are aware, have the skills to teach them. *Few teachers demonstrate to their students their own personal strategy use. In general, teachers are not aware of the importance of these skills. The fact that there is such little data leads to the assumption that strategy instruction is not a general classroom practice. [Italics added] (CEC, 2002, p. 7).*

Our collective interest across our two interrelated studies was to investigate the implementation of CSI/SBI/SCL in everyday classrooms.

CSI/SBI/SCL in Context of the Research Literature and the Everyday Classroom

Although each of these approaches, CSI/SBI/SCL, has a history both of educational research and classroom implementation, as indicated above, the Council for Exceptional Children (2002) suggests that very little strategy instruction is employed within classrooms (in the U.S. anyway and probably here in Canada as well). Perhaps not unrelatedly, the genesis of these various strategic instructional approaches appears to be in the context of individualized, tutoring, "pull-out," special education practices. SCL, for example, began as Butler's dissertation research in which she tutored one-on-one adults with learning disabilities. Since that time, Butler has worked with other researchers to apply SCL in mostly middle years and secondary classrooms. More currently, Butler has advocated and demonstrated how SCL may be expanded for use in small group and classroom-wide instruction (see Butler, 2002 p. 87). Hutchinson's 1993 study using CSI was conducted in small groups with students with learning disabilities. Often SBI research within mathematics is conducted with students with learning disabilities either individually or in small groups (Jitendra & Beck, 1999; Jitendra et al., 2002; and Ping Xin, et al., 2005).

Research Questions

There are comparatively fewer studies that look at CSI/SBI/SCL in the inclusive classroom (an exception to this trend may be Owen and Fuch's 2002 study in which typical peers served as tutors for third grade students in mathematical problem solving, although the students with challenges had learning disabilities). There are fewer studies still that consider CSI/SBI/SCL approaches with students with other kinds of challenges than learning disabilities, e.g., the developmental or intellectual disabilities studied here. The following four research questions then guided our investigation for the first study carried out in the classroom of Kelly Howell Daziel:

1. Does schema-based instruction work in an inclusive grade four classroom?
2. How will students in an inclusive grade four classroom understand and utilize schema-based instruction?
3. How can I make word problems relevant for students?
4. How can my instructional practices as a teacher be inclusive in a grade four math class?

Our research questions in Lara Grismer's classroom were similar, although tailored to her environment:

1. Does teaching elements of CSI/SBI/SCL/CRA as whole group instruction lead to successful outcomes for students with learning challenges?
2. What specific strategies outlined in CSI/SBI/SCL/CRA are effective for grade nine math instruction?
3. Will CSI/SBI/SCL/CRA have a positive effect on students' attitudes toward higher-order math challenges?
4. As an educator, how easily implemented is teaching using the CSI/SBI/SCL/CRA strategies?

To be concise, we saw our two interrelated research projects as extending the literature in the following ways: we implemented CSI/SBI/SCL as whole group instruction instead of following a one-to-one tutoring model, within an inclusive classroom, and finally, with students designated as having challenges other than a learning disability.

Methodology

Action Research Framework

To answer these questions, we conducted two action research projects. One was carried out in Kelly Howell Daziel's inclusive grade four classroom, the other in Lara Grismer's secondary math class specifically for students with learning disabilities. Consistent with the mandate of the McDowell Foundation, we employed an action research framework. We were interested in the practical nature of applying strategy instruction in actual classrooms. As a team we met and discussed issues as they arose throughout these two projects. We reflected on how our own practice shifted throughout the projects, as well as the impact upon our students. Perhaps not incidentally, action research has quite a sustained history within inclusive/special education (see Babkie & Provost as but one example), and indeed, one of the most useful and most cited peer-reviewed journals in the field is Intervention in School and Clinic, which is designed primarily for action research studies in which actual teachers as researchers work in actual classrooms with actual students.

General Data Gathering

As a research team we used many techniques to gather data. First, each teacher maintained a Research Journal in which she documented her thinking regarding the CSI/SBI/SCL instructional phases of these projects. We determined and documented a baseline for each of the students with respect to their mathematics conceptual and procedural knowledge (the particularities of these baselines are explained in more detail in the subsequent sections). We also maintained all student outcome data that showed how students actually performed on relevant higher-order mathematical word problems. In addition, student participants with learning challenges were interviewed (with their consent) regarding the efficacy of the CSI/SBI/SCL and CRA instructional procedures. These interviews were tape-recorded and transcribed.

General Analytical Techniques

As a research team, we used the following techniques to analyze data. Kelly and Lara each read through the transcripts and examined all student baselines and intervention data. They also considered their Research Journals and field notes in their entirety. All data were coded and then categorized. As a team, we arrived at the results, which are primarily answers to our respective research questions for the first and second action research projects. These qualitative data analysis techniques are fairly standard.

The First Action Research Project:
Kelly Howell Daziel's Inclusive Grade Four Classroom

PARTICIPANTS (WHO)

Kelly's action research took place in a grade four classroom with 15 students, seven male and eight female. One of the students, Eric, has been diagnosed with Fetal Alcohol Spectrum Disorder. There was often an Educational Assistant in the classroom.

TABLE 1: PERFORMANCE LEVEL OF STUDENTS IN THE GRADE FOUR INCLUSIVE CLASSROOM

Student Pseudonym	Gender	Level
Eric	Male	Progressing
Kara	Female	Surpassing
Kent	Male	Surpassing
Quinton	Male	Meeting
Lindsay	Female	Progressing
Betty	Female	Meeting
Darla	Female	Progressing
Dorothy	Female	Meeting
Ben	Male	Meeting
Lyle	Male	Meeting
Cindy	Female	Surpassing
Ken	Male	Meeting
Ed	Male	Meeting
Leslie	Female	Meeting
Brie	Female	Meeting

Table 1 above indicates each student's level of performance on curriculum objectives using the descriptor given to parents in the report card. "Progressing" indicates that the student has shown progress in meeting curriculum objectives; "meeting" indicates that the student has met the curriculum objectives; and "surpassing" indicates that the student has met and surpassed the curriculum objectives. Curriculum objectives are those listed in the Saskatchewan Evergreen Curriculum for grade four mathematics.

Procedure

When (Research Timeframe)

The research took place from January 2007 to May 2007. The content was divided into four smaller units and taught in one-week blocks. The first three blocks were taught about one month apart from each other and the final block was taught immediately after the third unit. The decision was made to teach the content in one-week blocks to maintain the interest of the students and make the content more manageable for the students to learn. The different problem types were taught one at a time, one problem type per one-week block. The students had math class everyday and the period length for math ranged from 30 to 75 minutes.

The Content (What) – Teaching Word Problems

Solving word problems using schema-based instruction was the focus of what was taught to the students. Specifically, three types of problems were used: change, compare and group problems. The grade fours had completed a unit of addition and subtraction in October/November and had mastered the addition and subtraction skills needed to solve these problems successfully.

Prior to starting the problem solving units all students completed a nine-question pre-test to establish a baseline. The pre-test contained three problems, one of each type. Some problems required addition with and without regrouping and some required subtraction with and without regrouping. The variables used in the problems did not exceed three digits in length. The students were not given a time limit for completing the pre-test. They were asked to do their best and show any work that they did to solve the problems. All the problems on the test were read aloud to the students at the commencement of the pre-test, at which time the students had the written copy in front of them and could follow along. During the pre-test a student could request the teacher to reread a problem. If students requested further assistance they were prompted to do the best they could.

The results from the pre-test were used in comparison with the results from three-question post-tests at the conclusion of each unit. The results from the pre-test were also used in comparison with the results from a nine-question post-test at the end of the all the units.

Three different problem types were taught to the students: change, group and compare problems. These types were taught in three blocks that each focused specifically on one type of problem. The blocks were approximately one week long with three weeks between the end of one unit and the start of the next. Following the third unit a review unit was taught. Due to the repetitive nature of the way the problems were taught, the time between the units was needed to maintain student interest levels.

Each of the units on the different problem types followed the same pattern. The students were first taught to identify the problems, then represent the problems and finally focus on problem solution.

1. *Problem Identification*. Introduce the problem with all of the variables known and teach the parts of the problem and common characteristics.
2. *Problem Representation*. Introduce the schematic representation and how to map the information from the problem onto the representation. A visual cue card is introduced to the students.

3. *Problem Solution*. Once the information is mapped, what steps do we take to solve the problem? We found "T" for the Total and then we decide what operation to apply to solve the problem.

CHANGE PROBLEMS

Change problems infer a passage of time and are about one thing. Change problems include a beginning set, a change set and an ending set. The example given below may be referred to as a story situation because all of the variables are known.

> *Example: Mrs. HD had 16 paper fish in her fish bowl (beginning set). She lost 4 of them when she was in the gym (change set). Now Mrs. HD has 12 paper fish (ending set).*

Change problems were taught for one week and during that time students completed 18 change problems. At the conclusion of the unit a post-test with three change problems was given. These results were compared to the change problems on the pre-test.

Problem Identification. Change problems were introduced to the students in two class periods using story situations. Each set of the change problem was discussed using an example story situation with the entire class. Through class discussion the students decided that the best way to identify all of the sets in the problem was to highlight each set using a different colour. The students decided as a class which colours would represent each set. Then the entire class labeled two more story situations. With three story situations as examples, the characteristics of a change problem were examined and discussed.

The students were grouped in pairs and given the task of creating two story situations on large chart paper using the three examples as references. When each pair of students completed this task, they joined another pair. Together the two pairs identified the sets in the story situations by highlighting the sets using the predetermined colours. Each pair then presented their story situations to the entire class. During these presentations they identified the sets of a change problem.

The story situations used to introduce the students to change problems were created by the teacher and included students' names and interests. This approach was adopted purposely to gain student excitement and interest. The story situations that the students created also contained their names and interests. These story situations became the examples and change problems that the students solved for the duration of the unit. Refer to Appendix A for sample student-created problems.

Problem Representation. The next step for the students was to learn how to represent the sets that they were able to identify. This is done using a schemata representation (see Appendix B). A schemata representation is a graphic organizer that includes the sets of a problem and demonstrates the relationship between those sets. Transferring the data from the story situation or problem to the schemata representation will be referred to as mapping the information.

The schemata representation for a change problem was introduced to the students using an overhead and the original three example story situations. As an entire class we mapped the information while discussing the sets of the change problem. At this time a visual cue card that included an example of the schemata representation for a change problem, the colours to use to identify a set and the characteristics of change problems were distributed to each student. This cue card remained at their desk for use when solving problems.

After the introduction of the visual cue card, the students were given 12 change problems from the bank of story situations they had created during problem identification. This was the first time the students had encountered a change problem. Until now the students were using story situations with all variables known. They now had change problems with one variable unknown. As a class we discussed how to represent the unknown variable on the schemata representation. It was decided that a question mark would be used to indicate the unknown variable.

The students' task was to map the information from the 12 problems that they were given using the schemata representation after they had highlighted the different sets in the problem. For the first four questions the schemata representation was provided on the worksheet. For the remaining eight questions it was expected that the students would draw the schemata representation themselves.

Problem Solution. Finally the students were ready to solve the change problems that they had identified and represented. The first step in problem solution is to teach the students to find the total. The total is the set that has the greatest amount in a problem. In a change problem the total can be the beginning or ending set. The purpose of finding the total is to determine which operation is needed. If the total is unknown then students will add to find the solution. If the total is known then they subtract to find the solution. Once the students identify which set is the total they can mark it with a "T".

A cue card (see Appendix B) that contained information about finding the problem solution, finding the total and determining the operation was distributed to the students at this time to accompany the visual cue card. These two cards were kept at the students' desk held together with a binder ring.

Continuing with the 12-question worksheet, the students found the total and marked it with a "T". After the students had completed this task, they were ready to carry out the last steps of solving the problem, which included determining the operation and carrying out that operation.

To determine how the problems would be evaluated the students decided how each problem would be marked. After some discussion it was decided that each problem would be marked out of five. One mark was given for each of the following: mapping information correctly, finding the total, choosing the correct operation, performing the correct calculation and providing a sentence that explained the answer.

Throughout the change problem unit the variables in the story situations and problems were purposely lesser numbers (i.e. numbers less than 50) to facilitate a few factors. Manipulatives such as bingo chips were used to demonstrate a story situation or a problem and having lesser amounts in the problem made the use of manipulatives very manageable. If students needed to draw the objects in the problem for more understanding, lesser amounts also made this more manageable. Also, downplaying the variables may have allowed the students to focus on the three steps to solving change problems.

Group Problems

Group Problems contain at least two smaller sets and one larger set. The amount of an object does not change and there is a part-whole relationship. There is more than one object in a group problem but there is an association between the two or more objects, such as the fact that an apple and orange are both fruits.

> *Example: Tina had 9 tubes of lip smackers in her purse (larger set). Six tubes were glittery (smaller set) and the remaining 3 were fruit flavored (smaller set).*

Group problems were taught for one week, and during that time students completed 24 group problems. At the conclusion of the unit a post-test with three group problems was given. These results were compared to the group problems on the pre-test.

Problem Identification. Group problems were introduced to the students in one class period using a story situation. Each set of the group problem was discussed using an example story situation with the entire class. Through class discussion the students decided on the best way to identify the sets in the problem. It was decided, as was the case with the change problems, to highlight each set using different colours. During the initial introduction to group problems, it was discussed how group problems are different from change problems. Group problems have no passage of time, there is more than one object and the object amount does not change. Prior to having the students group into pairs to create their own story situations, the entire class brainstormed a list of items that could be written about. Because objects in a group problem have an association, the association could be clearly demonstrated by creating a list for the students. The students could also refer to this list when they were creating their story situations.

It became clear that the students had become very comfortable with the process of creating their own story situations due to their experience with change problems. Students demonstrated this increased level of comfort by creating story situations more quickly. They also began to create more difficult situations by increasing the value of the variables and including more than two smaller sets. The students expressed a degree of pride and confidence that they knew they would be able to solve more difficult problems.

Problem Representation. Problem representation for group problems was introduced using student-created problems and the schemata representation (see Appendix B). As an entire class we mapped the information from the problem to the schemata representation. The unknown variable was marked with a question mark. At this time a visual cue card that included an example of the schemata representation for a group problem, the colours used to identify a set and the characteristics of a group problem were distributed to each student. This card could be added to the binder ring with the previous cue cards for change problems and kept at the students' desks.

After the students were given the visual cue card, their task was to map the information from the 12 problems they were given using the schemata representation after they had highlighted the different sets of the problem. For the first four questions the schemata representation was provided on the worksheet. For the remaining eight questions it was expected that the students would draw the schemata representation themselves.

Problem Solution. The students were ready for solving the problem. The first step, as it was for change problems, was to find the total. To demonstrate that the larger set is always the total, a few example problems were demonstrated using bingo chips and shoeboxes. The bingo chips represented the objects in the problem and the boxes were used to represent the boxes in the schemata representation. The students continued to identify the total using a "T". In a group problem this is a simple process because the larger set is always the total. A cue card (see Appendix B) that contained this information and information on determining the operation was distributed to students at this time to be added to their binder ring of other cards.

Continuing with the 12-question worksheet the students found the total and marked the total with a "T". After the students completed this task they were ready to carry out the last steps of solving the problem which included determining the operation and carrying out the operation. The problems were evaluated using the same five-mark process that was established during the change problem unit.

Compare Problems

As the name suggests compare problems compare the difference in value between two sets. The amount stays the same, the object stays the same and the passage of time is not a factor. Compare problems have the first compared set, a second compared set and a difference set.

> *Example: Dale has 15 computer games (1st compared set) and Perry has 29 (2nd compared set). Perry has 14 more computer games than Dale (difference set).*

Compare problems were taught for a little more than a week, in which time students completed 18 compare problems. At the conclusion of the unit a post-test with three compare problems was given. These results were compared to the compare problems on the pre-test.

Problem Identification. Compare problems were introduced to the students during one class period. The example story situation was reviewed on the overhead with the entire class. Each set of the story situations was discussed with the students. Through class discussion the students decided on the best way to identify the sets in the problem. Again it was decided to highlight each set using different colours. During the class discussion the story situations were drawn on the overhead to demonstrate the relationships between the variables. At this time we were able to discuss the differences between the compare problems, change problems and group problems.

With the examples as a reference the students were grouped in pairs and set to the task of creating at least two compare story situations. The students worked in assigned pairs, and often a struggling student was paired with a stronger student.

It appeared that as the students were working at creating the compare problems, they were struggling with understanding the relationships between the sets. It could have been the complexity of the vocabulary. For example the terms "compared" and "difference" sets are not as concrete as "beginning," "ending," "change," "smaller" or "larger" sets as found in the other problem types. As a result the entire class worked through some examples and actually drew out the story situations as well as visualizing the relationships. This appeared to develop some clarity for students.

The story situations that the students created built a large bank of problems that were the type of problems that we needed and were relevant to the students. Having the students give all variables allowed them to demonstrate their understanding of the relationship between the sets. This was accomplished when the variables in the story situation correlated. When the problem worksheets were created for the students, the teacher had the ability to leave different sets unknown and move some of the sets around in the problem. The students' creation of the story situations helped solidify their knowledge of problem identification and created a bank of problems that the teacher could easily access.

Problem Representation. Problem representation was introduced, the schemata representation was shared with the entire class and sample problems from the previous day were mapped. As with the other problem types, the unknown variable was represented with a question mark. At this time the final visual cue card (see Appendix B) was added to the binder ring of cue cards for student reference. After the students were given the visual cue card, their task was to map the information from the six compare problems that they had created during problem identification. On this worksheet the first two problems had the schemata representations included. For the remaining four the students were expected to draw the schemata representation.

Problem Solution. The students were then ready to solve the problems that they had identified and represented. The first step, as it was for the other problems, was to find the total. In a compare problem the total can only be the first compared set or the second compared set. The students continued to identify the total using a "T". A cue card that contained this information and information on determining the operation was distributed to students at this time to be added to their binder ring of cards.

Continuing with the six-question worksheet, the students found the total and marked it with a "T". After the students completed this task they were ready to carry out the last steps of solving the problem, which included determining the operation and carrying out the operation. The problems were evaluated using the same five-mark process that was established during the change and group problem units.

Reviewing Change, Group and Compare

The students were proficient at solving the problems when they knew what type of problem they were solving. The type of problem had been obvious in the three units of study that were completed because the focus of each unit was exclusively on one kind of problem. The next challenge was to have the student identify the different problems when they were all presented together. To begin this process the entire class reviewed the three problem types. Then the students were divided into groups of three and given one of the three problem types to act out for the class. The class would then need to decide what kind of problem it was, represent it and then solve it. The students had access to their binder ring of cue cards for reference if needed.

The students then had the opportunity to practice their identification skills in small groups with a sorting activity. They were given 12 problems that included four of each problem type, and they were asked to sort the problems into change, group or compare. This task was completed in small groups of three. Again, the students were able to refer to their binder ring of cue cards. Once the problems were sorted into the appropriate groups, the students' task was to represent and solve the problems.

The students completed two nine-question worksheets after the problem types were reviewed again. It was indicated to the students while they were working how many of each problem type were on the worksheet.

In these activities problem solving became something that truly started for the students at the problem identification step. In the previous units of study they had been working with only one type of problem. With these tasks the students were working with all three at once. It seemed that after the task of identifying the problem was completed, they could easily continue with problem representation and solution.

The students created their own checklist to cue themselves when solving a problem. This checklist would potentially help to decrease the number of minor mistakes, such as calculation errors or incorrect recording of the equation. As a class the students brainstormed and came up with the following acronym to remind them to think about different steps when solving problems. They called it ROSSI.

R – Read

O – figure Out what kind of problem

S – Solve the problem

S – Sentence

I – check It over

The students then solved six more questions while using ROSSI. Once the students had completed any corrections on the review problem worksheets, they completed the nine-problem post-test, during which the students could use their cue cards.

Pre- and Post-Tests

The pre- and post-tests were all completed during math class in the grade four classroom. At the start of all of the tests, the questions were read aloud while the students had the written copy in front of them. The tests were read aloud so that a student's reading ability would not be a factor in the successful completion of the problem solution. At any time during the tests the students could ask the teacher to reread a problem for them. If students were to probe further, they were told to do the best that they could and show all of their work.

The problems that the students were given to solve only varied by characters or objects and variables from the pre-test to the post-test. The variables in each question would require the student to perform the same type of operation as the comparable question on the pre-test. For example, if a problem on the pre-test required a student to subtract with regrouping occurring in the ones and tens place, the same was required on the post-test.

Results

To reiterate, the purpose of this first action research project was to answer four questions about students and higher-order mathematical problem solving. The results given below are structured around these questions.

Does schema-based instruction work in an inclusive grade four classroom?

Schema-based instruction appeared to be very successful in an inclusive grade four classroom. The students had all of the strategies that they needed to solve word problems. On the pre-test the students solved 59% of the problems correctly and on the post-test they solved 81% of the problems correctly. The students demonstrated an increase in the correct problem solution.

The increase appeared to be a result of a decrease in the variety of errors that were made in the pre-test. On the pre-test eight different types of errors were made on 41% of the questions. On the post-test only three types of errors were made on 19% of the questions. Of the errors that were made on the post-test, almost half were comprised of students mapping the information incorrectly. The other two errors were recording the question incorrectly and calculating incorrectly. This decrease in errors can be attributed to schema-based instruction. The students were using a strategy that would work for them on a variety of questions. For a complete overview of the error analysis on the pre-test and post-test, refer to Appendix C, figure 1.1 and 1.2.

On the post-test every student used a schemata representation to solve the problem and 89% of the problems were represented correctly. During the pre-test it was not requested that the students represent the problem. The students were asked to show their work, and on analysis of the pre-test, 4% of the problems were represented using some form of graphic representation. This result demonstrates that the students were able to integrate the problem representation strategy. On the pre-test students were not required to identify the problems, but on the post-test 87% of the problems were identified correctly. For a complete overview of the analysis of problem steps refer to Appendix C, figure 2.

How will students in an inclusive grade four classroom understand and utilize schema-based instruction?

A group of seven students completed three problems using think aloud strategies while being tape-recorded before and after the problem solving units were taught in the classroom. The problems consisted of one of each problem type. During the post-test students were able to use their visual cue cards if needed.

A noticeable pattern emerged when analyzing the data. Prior to the units being taught, the students appeared to have one or two strategies that they could rely on to solve problems. The first of these was the key word strategy, i.e., a strategy of identifying key words such as "total" and "less than." These key words then indicate to the students which operation to apply to solve the problem. For some questions this strategy is quite effective but for others, the rule does not apply. It seemed that when students used this strategy they were zeroing in on only one small part of the problem and not examining the relationships between the variables in the problem. The other strategy, relied on by one student, Darla, was to draw to solve the problem. This strategy became difficult when the variables in the problem were greater, and as a result, she realized that she did not want to draw the hundreds of items in some problems; however, she did not have any other strategy to rely on.

Schema-based instruction appeared to give the students a strategy that could be used to solve the problem from beginning to end. Students who completed the think aloud during the post-test first identified the problem type, then they continued to the steps of problem representation and problem solution. All the students who completed the post think aloud problems shared a commonality not evident in the first think aloud. They all used the language of problem solving. They talked about problem types and finding the total. The students were comfortable using the new vocabulary and had apparently made it part of their problem solving strategies.

The students demonstrated increased confidence in approaching and solving the problems. Darla correctly identified the problems but made errors when she represented a problem on the schemata representation. Even though she made errors mapping the information, she still needed to interact with the variables and understand the relationships between them to properly identify the problem type. On the first think aloud she often had difficulties knowing where to start and what to do, and she demonstrated on the post think aloud that she was thinking about the variables in the problem and the relationship that they had with each other. A student like Darla just needed more time to make that strategy her own and gain more confidence using it.

On the post-test Darla was very worried about identifying the problems correctly. Although she did not identify six of the problems correctly, she found the correct problem solution for these six. This result demonstrates that she was making the schemata representation work for her. She understood what the relationship was between the variables and was able to make meaning of them. This understanding enabled her to find the correct solution in the absence of correct problem identification.

How can I make word problems relevant to my students?

(Response from Kelly Howell Dalziel)

Making word problems relevant to my students occurred in part out of necessity. The problem solving units completed required specific problem types. A main math resource such as a textbook was not used or available for the grade four class, so selecting problems from a textbook was not a possibility. Searching for specific problems that would fit into the change, group or compare categories was potentially time-consuming and frustrating. To suit my needs I created about fifteen problems of each type to use on the pre-test and post-test and introduce the students to the problem types. To generate enthusiasm among the students I used their names and their interests in the problems, being certain to include all of the students in the problems. The students were often very excited to see their names and interests included in the problems.

When the problem types were introduced and the students were learning about problem identification, the students wrote story situations. These story situations were used to create problem worksheets for the students. The students were very excited to see their problems being used and would often tell a peer with pride that they had written a particular problem. As the students realized how the story situations they wrote would be used, they became more motivated to create them.

Making word problems relevant to students by using their names and interests provides benefits other than motivation. For the students to connect with the problem and interact with the variables in the problem, they need to have background knowledge about the variables in the problem. This background knowledge allows a student to think about the relationships between the variables, which then enables the student to conceptualize the problem situation, making problem identification and representation more meaningful.

How can my instructional practices as a teacher be inclusive in a grade four math class?

(Response from Kelly Howell Dalziel)

Schema-based instruction gave the entire class an opportunity for success. The confidence levels of all students appeared to increase. Their increase in confidence was evident when analyzing how the students were solving problems. On the pre-test 96% of all problems were attempted and work was shown on 76% of the questions. On the post-test 100% of all of the problems were attempted and work was shown on 100% of the questions.

Eric, a student that is diagnosed with Fetal Alcohol Spectrum Disorder, solved four of the nine (44%) questions correctly on the pre-test, and he solved five of the nine (56%) questions correctly on the post-test. Although it was a small gain, Eric showed growth in areas other than correct problem solution. On the post-test he identified 100% of the problems correctly. He also represented 44% of the problems correctly as opposed to 0% on the pre-test. The fact that Eric was able to identify the problems indicated he is interacting with the variables and understanding the relationship between them. For a complete overview of the analysis of Eric's pre-test and post-test, refer to Appendix C, figure 3.1 and 3.2.

Discussion

The use of schema-based instruction in an inclusive classroom changed the way I viewed teaching the subject of math. Prior to this project I often taught the basic operations in isolation which I believe did not make the skills being taught relevant to the students. Teaching problem solving in this way helped me accomplish two important goals with my students. First, it demonstrated to the students that there is an application for the skills being taught such as addition and subtraction. This made the skill more meaningful for the students. The second goal that was accomplished was the students were examining the relationships between the variables in the problems. Schema-based instruction gave the students the scaffolding they needed to successfully solve problems.

The Second Action Research Project:
Lara Grismer's Grade Nine Math Classroom

Participants (Who)

Lara's action research focused on grade nine math students who all had specific learning disabilities. The research started with eight participants; however, due to attendance issues, data were collected on only six. Each of the six students had either low average or average intellectual ability and had a specific learning disability in the area of mathematics and/or reading. Out of the six students, one was a female, and five were male.

Student Pseudonym	Age and Sex	Area of Difficulty
Hustin	14 – Male	- diagnosed LD - math facts - math reasoning - fine motor - spelling - written language
Jack O'Neil	15 – Male	- diagnosed LD - fine motor - spelling
Jay	14 – Male	- diagnosed LD - math facts - math reasoning
Marry	14 – Female	- diagnosed LD - math facts - math reasoning
Matt Skitzelfritzal	14 – Male	- diagnosed LD - fine motor - math reasoning - spelling - reading comprehension
Tony Forbes	14 – Male	- diagnosed LD - reading comprehension - math reasoning - written expression

**Note: Hustin, Jack, Jay, Marry, Matt, and Tony were the pseudonyms chosen by the students for this research project.*

PROCEDURE

WHEN

This research project took place during the 2006-07 school year. The literature review was completed during the summer proceeding the 2006 school year. Parental permission slips were attained during the first semester. Pre-tests, post-tests, interview questions and lesson plans were also prepared during the first semester. The instructional strategies and data collection began at the beginning of March 2007 and continued until the beginning of April. This method of teaching and data collection was taught as a math problem solving unit for the grade nine students. One hour per day was allotted for the student's math class. In that time the interviews, think aloud strategies, pre-tests, instruction and post-tests took place.

BASELINE DATA

In order to establish a need for a new method of math word problem instruction, a survey was conducted as well as a pre-test on common math word problems.

Participants in the survey included six out of eight students in the Math 9 Class for students with learning disabilities. In response to the question, "How do you feel about doing word problems?" the students had varied responses. Hustin felt, "They are ok but I have a little bit of trouble." Jack stated word problems are, "Okay." Jay said, "Sometimes they are easy and sometimes they aren't." Marry shared that she, "Dislikes them" and said, "They are very difficult." Matt said that he, "Thinks that they are stupid and dumb." Tony was the only student who stated that he, "Loves them and does not find them hard at all." Overall, most of the students had a negative opinion of word problems.

After the initial interview led us to believe that most of the students in the grade nine math class did not enjoy math word problems, we had reason to believe a new way of teaching math word problems was needed. As a second step, a large pre-test was administered to the students to assess their previous knowledge and guide instruction in the area of math word problems.

The pre-test included 12 questions, three from each of the typical math nine word problems from the MathPower 9 textbook. The number that each student solved correctly was as follows: Hustin – 1, Jack – 1, Jay – 0, Marry – 0, Matt – 1, and Tony – 4. The students' answers were further analyzed in order to facilitate more valuable comparisons between pre-test and post-test data. The answers were examined using the following categories: "set up wrong equation," "incorrect calculation," "attempted not complete," "partially complete," "represented the problem," "stated the facts," "correct or partially correct," "not attempted," and "recorded answer without showing work."

Set up wrong equation. Placement in this category meant the student tried an equation or formula that was not correct. For example, the student said perimeter of a rectangle is equal to length plus width instead of equal to two times length plus two times width. Out of the 12 questions on the pre-test, the students "set up the wrong equation" the following number of times: Hustin – 0, Jack – 1, Jay – 2, Marry – 0, Matt – 0, and Tony – 4.

Incorrect calculation. In this category, the student set the question up properly, but made a mistake in doing the calculation. For example, the student knew the formula for perimeter but then multiplied incorrectly and therefore got an incorrect answer. Out of the 12 questions, the students made an "incorrect calculation" the following number of times: Hustin – 0, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0.

Attempted but incomplete. This code indicated that the student tried a few strategies to answer the question but did not record an answer. The category did not include instances in which the students just recorded an answer without showing work. Out of the 12 questions the students "attempted but did not complete" the following number: Hustin – 0, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0.

Partially complete. This category meant the student answered part of the question correctly, but did not entirely finish the question to get it correct. For example, if the question asked for four consecutive numbers and he/she answered for the first number and forgot or did not say what the other three numbers would be. Out of the 12 questions the students "partially completed" the following number: Hustin – 0, Jack – 1, Jay – 1, Marry – 0, Matt – 0, and Tony – 0.

Represented the problem. To receive this code the student had divided the problem into small steps and used some type of symbol to distinguish between the parts of the word problem. For example, when the student was ready to calculate the answer, he/she might have drawn a calculator or written the word "calculate." Out of the 12 questions, the students "represented the problem" the following number of times: Hustin – 0, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0.

Stated the facts. In this category the student recorded the important facts from the problem before attempting to solve it. Out of the 12 questions the students "stated the facts" the following number of times: Hustin – 0, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0.

Correct or partially correct. A total calculation was made of questions that were "correct" or "partially complete" in order to get a better understanding of how the students understood the word problems. Out of the 12 questions, the students got "correct or partially correct" answers the following number of times: Hustin – 1, Jack – 2, Jay – 1, Marry – 0, Matt – 1, and Tony – 4.

Not attempted. Students fell into this category if they left the question completely blank and did not attempt any strategies. Out of the 12 questions, the students "did not attempt" the problem the following number of times: Hustin – 7, Jack – 4, Jay – 0, Marry – 9, Matt – 7, and Tony – 2.

Recorded answer without showing any work. This category meant the student guessed at an answer or did not show how the answer for the word problem was determined. Out of the 12 questions, the students "recorded an answer without showing work" the following number of times: Hustin – 5, Jack – 6, Jay – 10, Marry – 3, Matt – 5, and Tony – 2.

The students struggled with the content of the pre-test. Taken together the interviews and the pre-test results made it clear that all students in the grade nine math class were in need of the implementation of new strategies to learn math word problems.

Content and Strategies
(Teaching Mathematical Word Problems Using CSI/SBI/SCL/CRA)

(Described by Lara Grismer)

For this research project, math word problems were taught to students with learning disabilities using techniques from Cognitive Strategy Instruction (CSI), Schema-Based Instruction (SBI), Strategic Content Learning (SCL), and Graduated Instructional Sequencing moving from Concrete to Representational to Abstract (CRA). The four different types of math word problems taught were ones that students in ninth grade mathematics typically find difficult: problems containing consecutive numbers, problems containing a perimeter aspect, problems concerning coins and dollar value, and word problems that require equations to solve. (A bank of sample problems may be found in Appendix D).

Throughout the teaching of the math word problems in the grade nine math class, various strategies from Cognitive Strategy Instruction, Schema-Based Instruction, Strategic Content Learning and Concrete Representational Abstract were used. They included:

- Teaching different types of word problems in isolation.
- Pre-teaching math vocabulary by making cue cards.
- Allowing students to work together to determine how they should set up word problems.
- Allowing students to work together to determine how to represent the parts of word problems.
- Allowing students to work together to determine how word problems should be graded.
- Teaching concepts from concrete to representational to abstract.
- Using think alouds to model how to think your way through word problems.
- Having students use think alouds to verbalize their thoughts.
- Using *Guided Questions* to lead students through difficult word problems.
- Incorporating the classmates' names into homework assignments and test questions.

As noted earlier, the four specific types of grade nine math word problems used in this research were taught as part of a word problem unit. All interviews, pre-tests, post-tests and activities were conducted as part of the students' one-hour daily math class. Within the large pre-test names of the students were used within the questions to spur their interest. Student names were also incorporated into all assignments and tests created by the teacher.

To ease students' apprehension of the large pre-test we started with a fun hands-on activity to learn the vocabulary of word problems. Each student was given a list of words that were synonymous with "add, subtract, multiply, divide, and equal to." Together as a class we created flash cards that had the math terminology synonym on one side and the signs for "add, subtract, multiply, divide, or equal" on the back. The students then had their own flash cards to use as a reference for the unit. Knowing the math vocabulary or "background knowledge" is a strategy outlined in the Cognitive Strategy Instruction. (Sample flash cards are found in Appendix E.) After this lesson the students were given their large pre-test.

The second class activity was to determine how to set up a math word problem properly. The class worked together with the teacher to come up with a strategy and steps for attacking a word problem. The first step the class decided on was to figure out what the word problem was asking the student to do. Second, the students determined the important facts from the word problem. The third step was to decide on the appropriate operation to use in setting up an equation to solve the word problem. After the word problem was solved, the students decided they should do a check to see if the answer was reasonable. The final step was putting the answer in a sentence, making sure that the student gave an appropriate answer to the question asked by the word problem. This guided teaching strategy to initiate group decision is outlined in the Strategic Content Learning approach (see Butler, 2002).

In order to represent the different parts or steps to the word problems, the students brainstormed many different symbols or words. As a group they shortened the steps to "problem," "facts," "calculate," "check," and "sentence." For the "problem" step, they decided on either a capital letter "P", the word "problem" or a question mark. For "facts" the students used a capital letter "F", the word "facts," or the symbol of a light bulb. For "calculate," they used a capital letter "C", the word "calculate," or the symbol of a calculator. "Check" was most commonly symbolized by a check mark; however, some students still preferred the capital letter "C" or the word "check." "Sentence" was represented as either the capital letter "S", the word "sentence," a dollar sign (because this symbol contains an altered letter "S".), or a dot (the period symbol). This method of representing word problems with pictures and symbols is part of the representational phase of both Schema-Based Instruction and Cognitive Strategy Instruction.

After the class decided how to set up and represent word problems, it was a natural progression for them to decide how our class' word problems should be graded. Together we decided to give one mark for representing and recording the problem, one mark for representing and recording the facts, one mark for representing and setting up the correct equation to solve the problem, one mark for representing and getting the correct answer, one mark for each alternate answer (if the question asked for more than one solution), one mark for representing and checking the answer, and one mark for recording the answer in a sentence. This guided teaching as a group decision is also part Strategic Content Learning (Butler 2002) (see Appendix F).

After the students had a basis for understanding how their word problems should be completed, they were ready to learn about think alouds. This strategy has the teacher verbally express every step of solving the word problem. No steps or reasons for the operations used are left out. This modeling strategy is outlined in Cognitive Strategy Instruction and was used throughout the problem solving unit whenever instruction was required. Also, the students were encouraged to think aloud about their word problems throughout the unit.

During math class I was able to model the think aloud method of expressing knowledge. I went through a number of problems on the board and thought aloud about each step of the problem. With relatively easy math problems on the board, the students took turns thinking aloud about how they were able to achieve answers to the problems. The class worked cooperatively to learn the technique. When students skipped a step, I would ask, "How do you get that answer?" Then they would have to go back and verbally explain each step. After the students were comfortable thinking aloud with math word problems, I presumed they were ready for our think aloud pre-test.

PROBLEMS CONTAINING CONSECUTIVE NUMBERS

The students were very nervous about taping themselves thinking aloud and did not feel prepared to take part in this process. Therefore, to build their confidence on one of the pre-test questions, I started to teach the students consecutive number problems using the concrete, representational, abstract strategy (CRA). This best-practice strategy is found in all three of the CSI, SBI, and SCL models. The concrete instruction used pennies on the students' desks so that they could visually absorb how consecutive numbers are related to each other. During this time the students worked in pairs, with the students getting their partners to figure out what their two or three consecutive numbers were by giving hints. For example, they would tell their partners what all two or three consecutive numbers added up to and how many numbers there were. After the concrete instruction was completed, we moved to representational instruction.

The students worked out how they could represent the first consecutive number, deciding that it should be "x". After working with the pennies, they noticed that with consecutive numbers, you only have to add one to get the next number, so the second number would be represented as "x + 1". The third number would then have to be "x + 1 + 1" or "x +2". The students quickly caught on that they could use this representation in their math assignments. While I let the students practice their knowledge of consecutive numbers, I was able to persuade the more confident students to take part in their think aloud pre-test, which was conducted as a one-on-one recording. I was there briefly to get the students started and did not listen to their verbalizations of their math word problems. As each student took his/her turn he/she became more and more comfortable. I was able to tape all students.

The day after the think aloud taping, the students worked on consecutive number problems as an assignment. They practiced representing their consecutive numbers using word problem solving steps that the class had composed. Throughout this class lesson I was able to help the struggling students by using the directed questions strategy as outlined in the Strategic Content Learning approach. This strategy uses specific questions to direct the student through the word problem towards the correct answer. The questions gave the students clues about the steps required to solve the word problem without actually telling them what to do. For example, if a student said she could not start the word problem, I would ask, "What is the first step to solving a word problem?" The student would know that she needed to represent key variables using a symbol and determine what the word problem was asking (See Appendix G for a complete sample of directed questioning). Directed questions are geared to the specific needs of the individual student. After this one-hour math class the students were comfortable with consecutive number word problems. The following day the post-test for consecutive number word problems was administered.

PERIMETER PROBLEMS

After consecutive numbers were studied, we moved on to perimeter questions. This type of problem was taught in the same manner as the problems containing consecutive numbers, moving from concrete to abstract. The purpose of the concrete lesson was to learn what the word perimeter meant. After a simple explanation, the students used rulers and metre sticks to measure the perimeter of many objects in the room, including desks, books and the chalkboard. The students were then instructed to work in pairs, and one person from each pair was to measure the perimeter of a triangle in the room. That student was to tell his/her partner the perimeter of the triangle and the measurement of two of the sides of the triangle. The partner had to figure out what the third side must be. Each partner took a turn at calculating the missing side of a triangle.

After this activity the students were given a more representational activity. They were told that a rectangle in the room had a perimeter of 30 cm and its length was 5 cm longer than its width. They were instructed to find the rectangle's width. As a group they decided to represent the width of the rectangle as "x" and the length as "x + 5". After drawing a rectangle and labeling all four sides, the students worked out a formula to figure out the perimeter (Perimeter = x + x + "x + 5" + "x + 5"). The students were then able to solve for "x" and determine the width of the rectangle (5 cm). After these exercises, the class came to the conclusion that the abstract formula for perimeter of a rectangle is P = 2L + 2W. This going from concrete to abstract is an example of Cognitive Strategy Instruction.

After this class the students were able to complete perimeter practice word problems. During the practice time, directed questions were used to help struggling students. The day following the practice problems, the post-test was administered. Some students forgot the formula for perimeter.

Coin Problems

I followed the teaching of perimeter questions with coin and money problems, which are typically the most difficult problems for students to understand. The first step was to learn the difference between the number of coins and value of coins using actual coins on the students' desks. When the students held up two nickels, I would ask, "How many coins are you holding?" Then I would ask, "What is the value of the coins?" We continued this with various numbers of coins with different values until it became obvious that there is a relationship between number of coins and the value of coins. An abstract formula was then derived to relate number and value of coins (value of coins = number of coins multiplied by the value of the specific coin).

To make the lesson more representational, the students then worked with partners. Each student chose a combination of dimes and nickels without telling his/her partner. The student gave the total value of the coins (e.g., 55 cents) and said the partner would have to figure out how many dimes and nickels were hidden under the student's paper. Then the students each told their partners how many more dimes or nickels they had compared to the other coin (e.g., I have one more dime than nickel). After these clues were exchanged, the class determined we should represent the unknown coin amount with "x". Then, as they did with the perimeter questions, the students represented the number of dimes/nickels (whichever they had more of) as "x + __" (e.g., x + 1). The students, with guidance from the teacher, came up with a formula to figure out the number of coins using the information given (value of coins = number of dimes(value of a dime) + number of nickels(value of a nickel)). For example, the resulting equation might state: 55 = (x + 1)(10) + x(5). The students could then solve this equation to find that x = 3, so there are 3 nickels and 4 (one more) dimes.

This representational example was more difficult and the students required more time to work through their coin word problems. Directed questions were used to guide the students through their coin word problem practice sheets. After the practice sheets were completed, the students did their post-test on coin word problems.

Setting Up Equation Problems

The last type of word problems taught to the grade nine students involved setting up equations to solve the problem. Like the other three types of problem this type was taught starting with the concrete and moving toward the abstract. For these word problems, knowing the math vocabulary was important. (It is necessary to note that just memorizing math vocabulary is not an adequate strategy to solve math word problems; this was simply the first step to learning this type of word problem.) As a class we spent some time reviewing the vocabulary flash cards.

After this review, the students each took four pencils and three pens and laid them on their desks. I told the students that the cost of four pencils and three pens was nine dollars and 75 cents. Also, they were told a pen is three times as expensive as a pencil. The students were to figure out how much a pen and pencil cost. To start working on this example problem, together they changed the words into a math sentence: cost of 4 pencils + cost of 3 pens = $9.75. The students realized they needed to represent the cost of the two unknowns (pencils and pens). So we represented the cost of a pencil as "x", and the cost of a pen as "3x". Then we were able to abstractly represent the entire equation: (x)(4) + (3x)(3) = 9.75. After solving for "x" the students were able to figure out the cost of a pencil was $0.75, and a pen was three times that or $2.25.

After a few group examples the students were ready for their practice problems. Directed questions were once again used to prompt students who were having difficulty with the word problems. The post-test for problems in which setting up equations was necessary took place after the students were comfortable with the practice problems. The think aloud post-test took place after all of the word problems were completed. The students were more confident about participating in the taping because they were more confident in their ability to think aloud. The purpose of the pre- and post- think aloud tests was to hear from the students how their thinking and problem solving methods had improved during the unit. No direction was given to the students during their think aloud post-test. After the think aloud post-test was completed, the students wrote their large post-test that encompassed all four types of word problems. This large post-test was essentially an exact copy of the large pre-test given before instruction, except the numbers had been changed.

Results

At the risk of repeating ourselves, we take this opportunity to remind the reader of the research questions, and again, the results given below are structured around these questions:

Does teaching elements of CSI/SBI/SCL/CRA as whole group instruction lead to successful outcomes for students with learning challenges?

To determine the effectiveness of the implemented strategies, the large post-tests were examined and compared to the large pre-tests completed before instruction of the unit began. The post-tests were graded and further analyzed in the same way as the pre-tests. On the post-test the number of questions that each student answered correctly was as follows: Hustin – 1, Jack – 5, Jay – 6, Marry – 6, Matt – 11, and Tony – 11. The number of times out of 12 that the students "set up the wrong equation" was: Hustin – 1, Jack – 7, Jay – 1, Marry – 0, Matt – 0, and Tony – 1. The number of times out of 12 that each student performed the "incorrect calculation" was: Hustin – 5, Jack – 0, Jay – 0, Marry – 4, Matt – 1, and Tony – 0. The number of times out of 12 each student "attempted but did not complete" the question was: Hustin – 5, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0. The number of times out of 12 that each student "partially completed" questions was: Hustin – 0, Jack – 0, Jay – 5, Marry – 5, Matt – 0, and Tony – 0. The number of times out of 12 that each student "represented the problem" by breaking it into steps was: Hustin – 12, Jack – 12, Jay – 12, Marry – 12, Matt – 12, and Tony – 12. The number of times out of 12 that the students "stated the facts" was as follows: Hustin – 11, Jack – 12, Jay – 12, Marry – 12, Matt – 12, and Tony – 12. The total of problems "correct or partially correct" for each student out of 12 was: Hustin – 1, Jack – 5, Jay – 11, Marry – 11, Matt – 11, and Tony – 11. The number of questions out of 12 "not attempted" for each student was: Hustin – 0, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0. The number of times out of 12 that each student "recorded an answer without showing work" was: Hustin – 0, Jack – 0, Jay – 0, Marry – 0, Matt – 0, and Tony – 0.

Comparing the data from the large pre-test to the large post-test, it is evident that all students experienced positive growth. Some significant growth in individuals is worth mentioning. For example, on the pre-test Matt did not attempt seven of the 12 questions (58%) and got only one correct (8%), while on his post-test he attempted all questions (100%) and got 11 (92%) correct (see Appendix H, figure 1). Looking at the number of correct or partially correct answers for Marry, Jay, and Matt, they all made a significant improvement from the pre-test to the post-test. Marry (see Appendix H, figure 2) had zero correct or partially correct (0%) on the pre-test and then obtained a score of 11 on the post-test (92%). Jay (see Appendix H, figure 3) and Matt both started with one correct or partially correct (8%) answer on the pre-test and scored 11 (92%) on the post-test. All students made significant gains in the areas of representing the problem and stating the facts to solve the word problems (see Appendix H, figure 4). Another positive trend in the analyzed data was that all students attempted every question and showed their work on the post-test (see Appendix H, figure 5).

Overall when comparing the pre-test to the post-test results, the numbers of "correct answers," "set up the wrong equation," "incorrect calculation," "attempted but did not complete," "partially completed," "represented the problem," "stated the facts," and "correct or partially correct" all increased in favour of the post-test results. *All of the increased values were positive and showed growth in the area of math problem solving, even though some of the categories sound negative.* The reason for a positive increase in "setting up the wrong equation" was because the student was at least attempting to write an equation in the post-test. An increase in "incorrect calculation" is positive because the student was on the right track to solving the question and must have had the equation set up properly. "Attempted but did not complete" shows that the student had tried to complete the question instead of giving up. An increase in "partially completed" was positive because the student answered part of the question correctly and just needed to use that information to fully complete the problem.

The pre-test versus post-test results for the number of problems "not attempted" and for which the student "recorded an answer without showing work" decreased dramatically. These results were very exciting and supported the expected result of the new math word problem instruction. All students attempted every question and showed work in their attempts to solve the word problems. Please see Appendix H, figure 6 for the complete graph of pre-data versus post-data results demonstrated in percentages.

What specific strategies outlined in CSI/SBI/SCL/CRA are effective for grade nine math instruction?

Throughout the teaching of math word problems, various strategies from Cognitive Strategy Instruction (CSI), Schema-Based Instruction (SBI), Strategic Content Learning (SCL), and Concrete Representational Abstract (CRA) were used. Teaching different types of word problems in isolation allowed the students to focus on and understand each type of word problem before learning new types. Pre-teaching math vocabulary and making cue cards helped the students recognize math language words throughout the chapter that were foreign to them. Allowing the students to determine as a class how to set up and evaluate their word problems gave them a sense of accomplishment and ownership. Encouraging the students to decide what symbol they wanted to represent each part of the word problem also gave them a sense of ownership. Teaching with the Concrete, Representational to Abstract method allowed the students to experience why the word problems were set up and solved in a certain way. The think aloud modeling from the teacher let the students hear why a problem was being solved in the manner it was. The think alouds done by the students in class enabled them to hear how they solved the problems. The Guiding Question strategy helped students solve the word problems with a sense of personal accomplishment because they were not told how to do the questions. Using the students' names in the word problems made the problems more interesting and personal. Therefore the students became more engaged in problem solving. Overall, specific strategies from each teaching method were effective for students with learning disabilities.

Will CSI/SBI/SCL/CRA have a positive effect on students' attitudes toward higher-order math challenges?

I saw a distinct difference in the attitudes of the students towards word problems at the end of the project. This difference was evident in the post-interview conducted with each student individually after the unit had been completed. When the students were asked what they had improved on, five out of six students replied that they now knew how to set up a word problem. Matt, who had originally said that word problems were "stupid and dumb" admitted that where he had most improved was in "liking them". When the students were asked what they would continue to do with word problems, all of them responded that they would continue to use the five-step method we created. Five out of six students enjoyed the hands-on activities, and the sixth, Jack, said he "wasn't really sure" about that strategy. When asked about the effectiveness of representing each part of a word problem with a symbol, five of the six students said it was helpful. Jack commented, "I felt like it took a lot of time, but oh yes, it did help". One other question asked was: "Would you recommend any of these strategies to friends struggling with math?" All students agreed that they would use some of our strategies; four said they would suggest the five-step method, and the other two said they would recommend the hands-on strategies. Essentially I received positive comments about the lessons and how the students really appreciated being involved in determining how they would be graded on word problems. The Cognitive Strategy Instruction, Schema-Based Instruction, Strategic Content Learning, and Concrete Representational Abstract strategies were a hit with the grade nine students.

As an educator, how easily implemented is teaching using the CSI/SBI/SCL/CRA strategies?

The cons of teaching math word problems with the CSI, SBI, SCL, and CRA strategies were minimal but worth mentioning. It did take longer to teach word problems using these different techniques. It also took considerable planning to prepare the hands-on activities. However, it was worth the effort because the students were engaged in the learning and had a positive attitude and experience. One other con about teaching in this way was that it was not possible to re-teach the entire lesson to a student who missed class. Therefore, the student who was absent did not receive the same quality of lesson.

Discussion

The research conducted using best-practice math word problem strategies in a high school setting to students with various learning disabilities was very beneficial. Students were engaged in the learning experience and negative attitudes towards word problems were shattered. Overall, CSI/SBI/SCL and CRA incorporate strategies that can be relatively easily implemented into a high school setting. The surveys and pre- and post- data demonstrated that when learning of word problems is enhanced, it is an enjoyable and satisfying experience for students and teachers.

General Discussion and Conclusion

Collectively, our research team enjoyed this positive experience in teaching math word problems as a form of action research. There were similarities and differences in the two classroom settings where the research was conducted. However, common benefits were found that related to growth of all students in the area of math word problems. Overall, the strategies of Strategic Content Learning, Schema-Based Instruction, and Cognitive Strategy Instruction were beneficial at both grade levels.

The teacher researchers, Kelly and Lara, were working in two classroom settings; an inclusive grade four classroom and a grade nine math class for students with learning disabilities. As a result, the approaches that the teacher researchers took in their respective classroom differed. The students in the inclusive grade four classroom were continuing to learn the basics of problem solving. Schema-based instruction was a successful way to facilitate the acquisition of problem solving strategies by grade four students. The students in the grade nine classroom were further along in their problem solving journey and were learning more advanced word problem strategies.

Generally the grade four students had positive attitudes towards math and were willing to learn math word problems. On the other hand, the grade nine students had negative attitudes towards word problems, and it took some convincing to get the students to trust and try the new strategies. This negative attitude was a barrier that made it difficult to implement math word problem strategies with the grade nine students. At the end of the teaching process, however, both the grade four and grade nine students had positive attitudes towards learning math word problems. Observation of this growth has made us curious as to what attitudes the grade four students will have towards word problems when they are in grade nine. Will their positive attitudes and a good start to their experiences with word problems carry over into future math classes? If so, these strategies could potentially help to break the barrier of negative attitudes towards math word problems.

Both teacher researchers appreciated the action research process because it gave them a chance to learn how to best teach word problems. It also gave them the opportunity to look at the individual student's previous knowledge and analyze each student's growth. It appeared that all students were thinking more about what the word problems were asking and looking for the relationship between the variables in the questions rather than just finding key words and guessing what operation should be applied. Also, both researchers found the visual strategies very beneficial for their students. The use of schemata representations in the grade four setting and the use of visual representations for word problem steps for the grade nine students seemed to suit the interests of the students.

Overall, the research team enjoyed the research process and found great benefit to teaching math word problems using strategies from Strategic Content Learning, Schema-Based Instruction, and Cognitive Strategy Instruction. Students in the participating math classes enjoyed the teaching strategies and the data collected appears to demonstrate that students increased their capacity to solve word problems.

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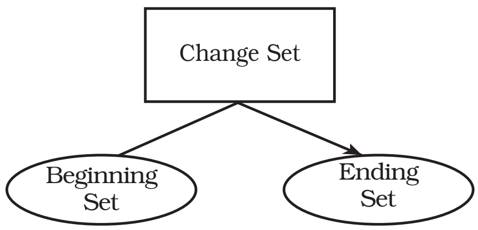
Appendix A:
Sample Grade Four Student-Created Problems

Changing Problems	1. Betty bought some oranges. Brie gave her 5 oranges. Now there are only 8 oranges left. How many oranges did Betty have to start with? 2. Leslie had 70 kitties. David bought some of the kitties. Leslie now has 11 kitties left. How many kitties did David buy? 3. There were 24 Volosa Raptors. 10 more came to play. How many Volosa Raptors were there in total? 4. There were 30 cookies at the bake sale. Lyle brought some more to sell. Then there were 56 cookies at the bake sale. How many did Lyle bring? 5. Ben, Eric and Kent caught lots of fish on their fishing trip last weekend. They ate 157 fish on the way back. There were 77 fish left. How many fish did they catch on their trip?
Group Problems	1. Kent had 100 toys. 50 of the toys were robots and the rest were turtles. How many turtles did he have? 2. Lyle scored 7000 goals in one game. Lyle shot some from the point and 3250 from his own end. How many goals did he shoot from the point? 3. The pet store had some cats. 270 of the cats had short hair and 231 had long hair. How many cats does the pet store have? 4. Kara has some baseball bats. 16 of them have blue handles and 23 have red handles. How many bats does Kara have? 5. In the abandoned house there were 70 rats. 4 of the rats had no diseases and the rest have diseases. How many rats have diseases?
Compare Problems	1. There were 400 emperor penguins and there were 250 mohawk penguins. How many more emperor penguins are there? 2. Brie has some flowers. Cindy has 77 flowers. Cindy has 29 more flowers than Brie. How many flowers does Brie have? 3. Dorothy has 100 happy face stickers and Brie has 99 happy face stickers. How many more stickers does Dorothy have? 4. Ken has 3000 cats. Preston has 2050 fewer cats than Ken. How many cats does Preston have? 5. Lindsay has 5 female calico cats. Ben has 3 fewer female calico cats. How many cats does Ben have?

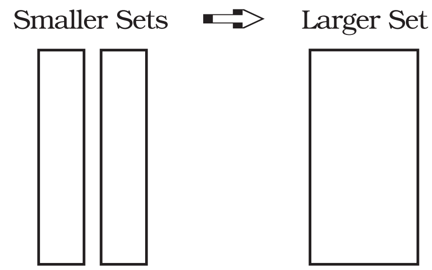
Appendix B:
Graphic Representations – Visual Cue Cards for Grade Four Students

Change Problem



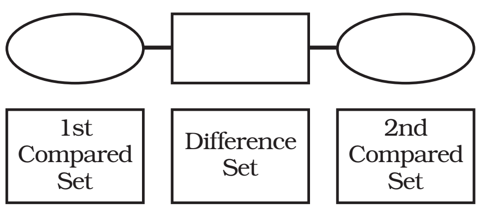
1. Change problems are about one thing.
2. From past to present.

Group Problem



1. The object amount does not change.
2. Time does not matter.
3. Not all the same object.

Compare Problem



1. Compare problems are about one thing.
2. Time does not matter.

T=Total
Finding the Total:

If the problem ends with **more** than it started with, then the **ending set** is the **total**.

If the problem ends up with **less** than it started with, then the **beginning set** is the **total**.

ADD or SUBTRACT?

When the **total is unknown, ADD** to find the total.

When the **total is known, SUBTRACT** to find the other amount.

From *Schemas in Problem Solving* (p.135) by S. P. Marshall, 1995, New York: Cambridge University Press. Representation adapted by permission.

APPENDIX C:
Grade Four Math Instruction Data

ERROR ANALYSIS

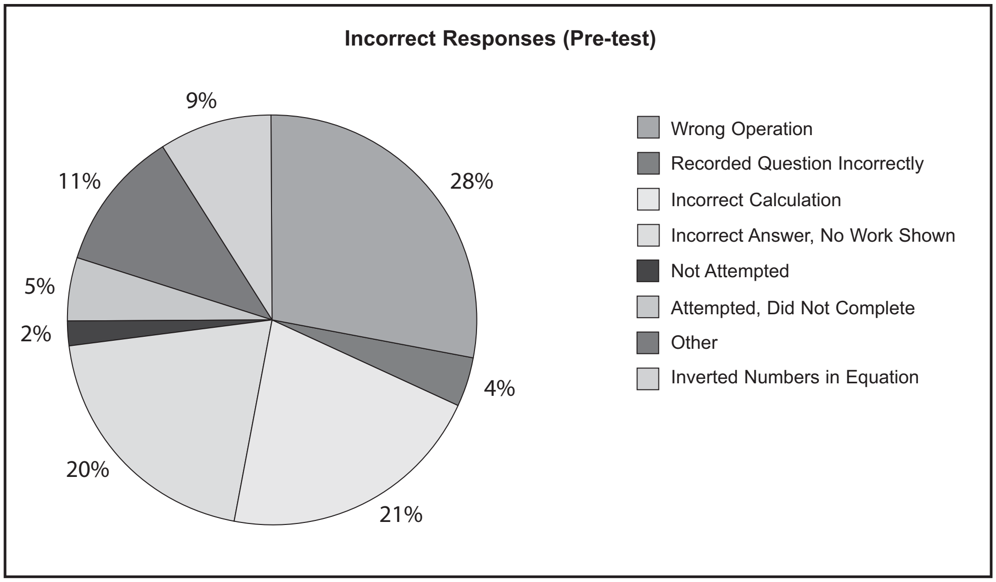


Figure 1.1

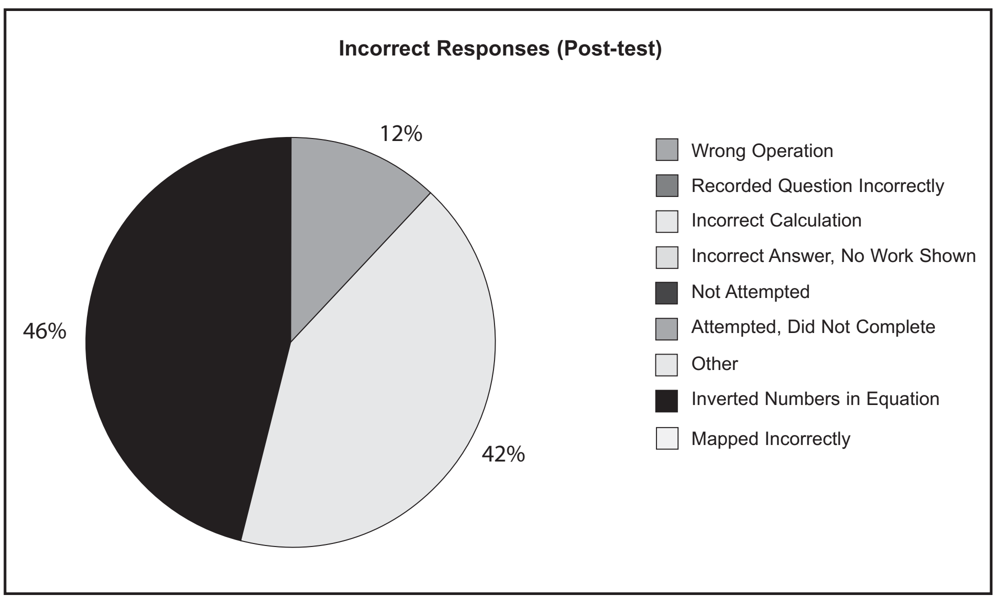


Figure 1.2

Analysis of Problem Steps – % Corrrect

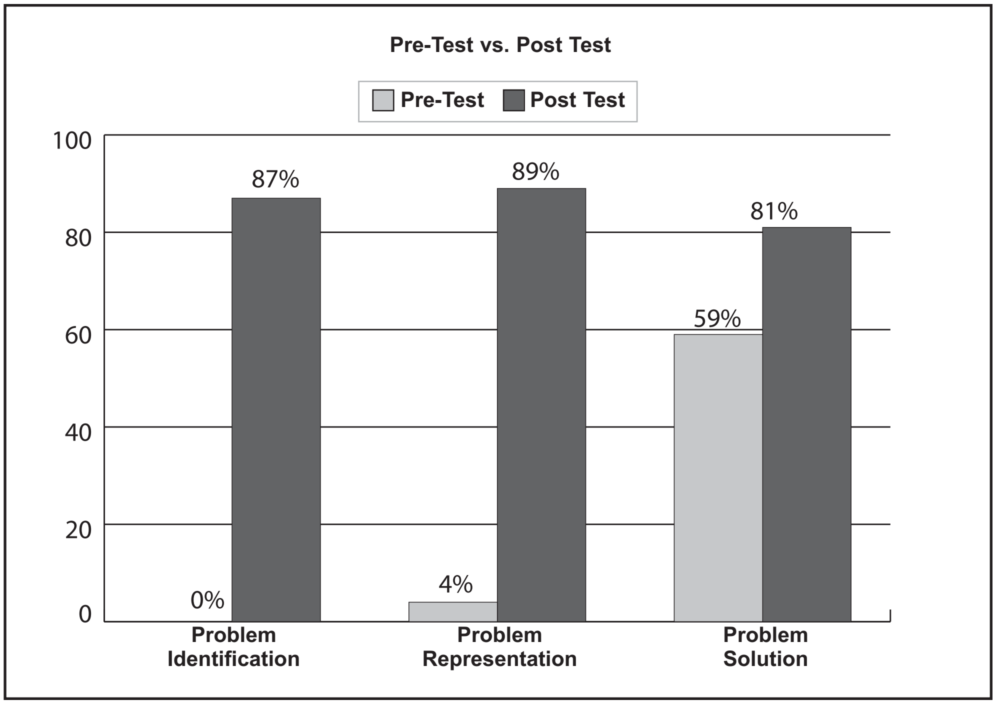


Figure 2

Analysis of Eric's Pre-test and Post-test

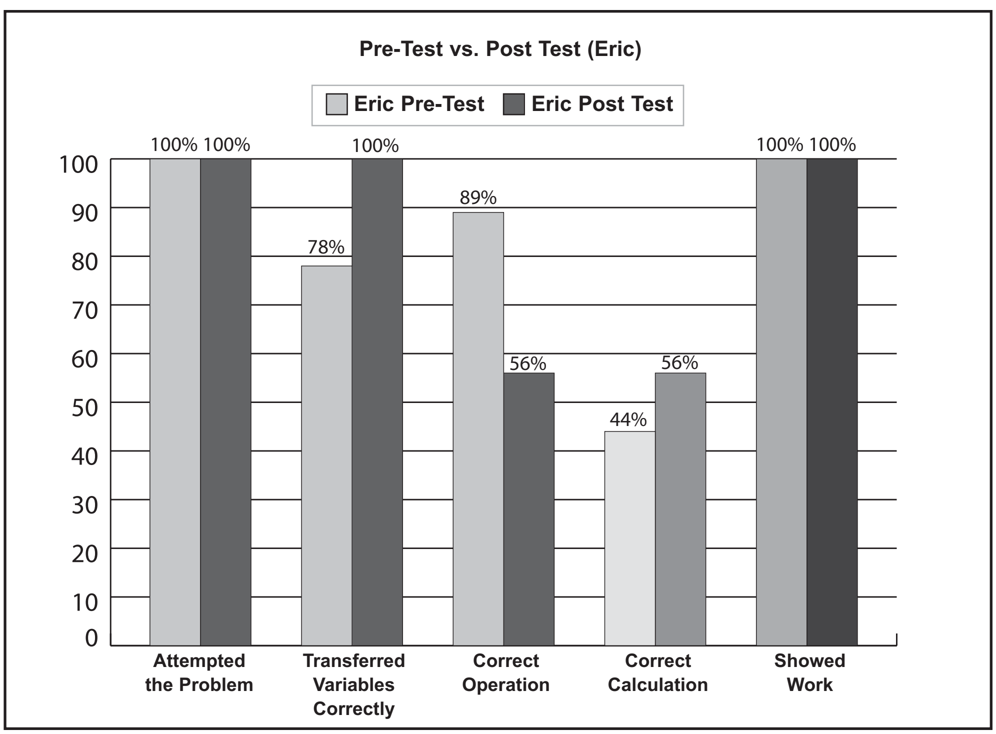


Figure 3.1

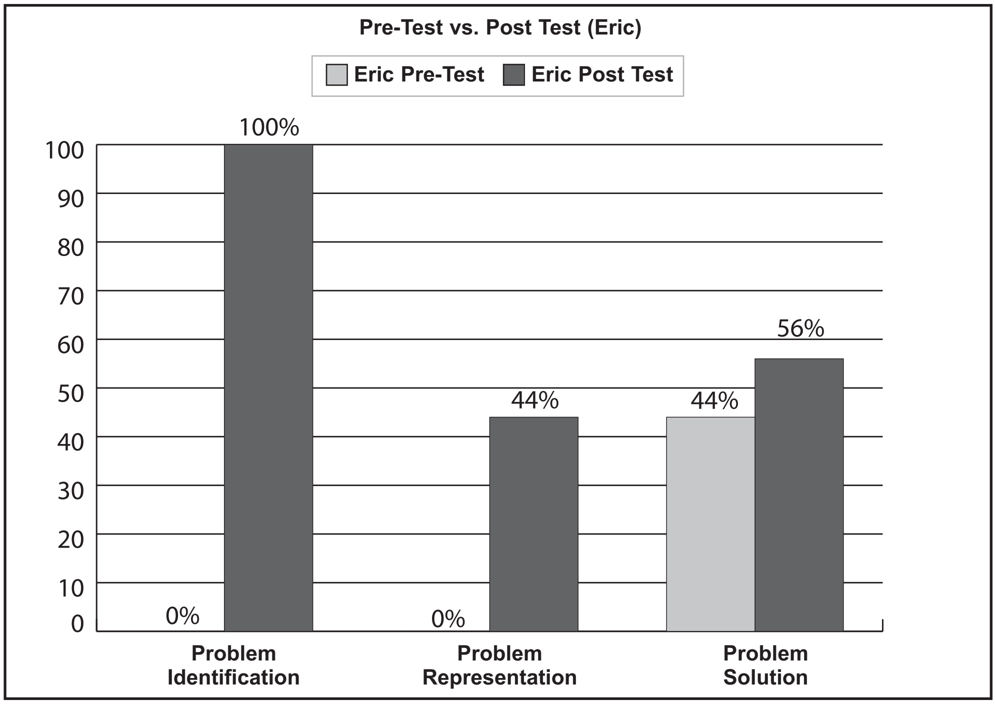

Pre-Test vs. Post Test (Eric)
Eric Pre-Test
Eric Post Test
100
90
80
70
60
50
40
30
20
10
0
100%
44%
44%
56%
0%
0%
Problem Identification
Problem Representation
Problem Solution

Figure 3.2

Appendix D:
Grade Nine Sample Word Problems

1. Consecutive Number Word Problems:
   a) The sum of 4 consecutive numbers is 50. Find the numbers.
   b) Matt picked 3 numbers. He hinted that the 3 numbers were consecutive and their sum was 78. Find the numbers.

2. Perimeter Word Problems:
   a) Find the length of Jack's room if the perimeter is 18 meters and the length is 3 meters longer than the width.
   b) Find the length of each side of an isosceles triangle if the perimeter is 14 meters and two of the side lengths are one meter longer than the third side.

3. Coin Word Problems:
   a) Marry was bored one Saturday, and calculated how much money went into a parking meter that only took nickels and dimes. She figured the value was $17.50 in nickels and dimes. She also counted that there were 225 coins. How many dimes are in the parking meter?
   b) Tony has $0.85 in nickels and dimes. He has 2 more nickels than dimes. How many nickels and dimes does he have?

4. Setting up Equation Problems:
   a) Hustin picked 2 numbers that nobody knew. The sum of the 2 numbers is 39. Twice the first number plus 3 times the second number is 101. Find the numbers.
   b) Jay figured he would then pick another 2 numbers. His hints were that one number is 5 more than the other number, and that 3 times the first plus twice the second is 30. Find the numbers.

APPENDIX E:
Vocabulary Flash Cards for Word Problems

Addition

PLUS	**ADDED TO**
INCREASED BY	**SUM**
GREATER THAN	**MORE THAN**

Subtraction

MINUS	DIFFERENCE
REDUCED BY	DIMINISHED OR DECREASED BY
GREATER THAN	MORE THAN

Multiplication

MULTIPLIED BY	TIMES
PRODUCT	OF
DOUBLE/TWICE	TRIPLE

Division

DIVIDED BY	QUOTIENT
SHARED EQUALLY	DIVIDEND
DIVISOR	PER

Equal To

EQUALS	IS
WILL BE	EQUIVALENT TO
THE SAME AS	

Appendix F:

Scoring Word Problems as Decided by Grade Nine Students

- 1 mark for representing and recording the "problem" – what the word problem is asking you to solve
- 1 mark for representing and recording the "facts" – state what "x" is equal to and any other relevant information
- 1 mark for setting up the correct equation to solve the problem
- 1 mark for "calculating" the correct answer (solving for x)
- 1 mark for each alternate answer (if the question asked for more than one solution)
- 1 mark for "checking" the answer
- 1 mark for recording the answer in a "sentence"

APPENDIX G:
Sample Directed Questions Used with Grade Nine Students

Below is a sample of a conversation used to direct a student towards solving a word problem using "directed questions." This sample conversation is not to be used as a script but as an example of the types of questions used to help the student. Depending on the student's answer to each question, the conversation changes and the teacher must adapt the directed questions to fit the need and ability of the student. As the questions are being asked the student is answering and then recording his/her work on his/her page.

Problem: The sum of 3 consecutive numbers is 105. Find the numbers.

Student: I have no idea how to solve this word problem.

Teacher: What are you looking to solve in this question?

Student: 3 consecutive numbers.

Teacher: How would you represent that?

Student: Use a question mark symbol and write down "find three consecutive numbers."

Teacher: What is the next step of word-problem solving?

Student: Record the facts, and I use a capital "F" to show that step.

Teacher: Great, what are your facts?

Student: I have three numbers and they are consecutive.

Teacher: How are you going to represent the smallest number?

Student: With an "x".

Teacher: How are you going to represent the other two?

Student: With an "x + 1", and an "x + 2".

Teacher: Great, what is the next step in solving the word problem?

Student: I need to calculate my answer by setting up an equation. So, I will draw a calculator to show what step I am on. Then I need to figure out my equation. I don't know how to set it up.

Teacher: Okay, what have we figured out so far?

Student: I am trying to figure out what three numbers add up to 105, and I have represented the numbers as "x", "x +1", and "x + 2".

Teacher: So how do you think that would look as an equation?

Student: I could write "x + x + 1 + x + 2 = 105", and then I could solve for "x". I get "x" is equal to 34. So, my first number is 34, my second number is 35, and my third number is 36. My next step is to check my answer and I represent that as a question mark. I don't remember how to do that.

Teacher: What are you “checking?”

Student: I want to make sure my three consecutive numbers add up to 105. Oh ya, so I could add the three numbers and make sure they add up to 105.

Teacher: Do they?

Student: Yes. So, I am almost done, I just need to write a sentence. I record this by drawing an “S” and writing, “The three numbers are 34, 35, and 36.”

APPENDIX H:
Grade Nine Math Instruction Data

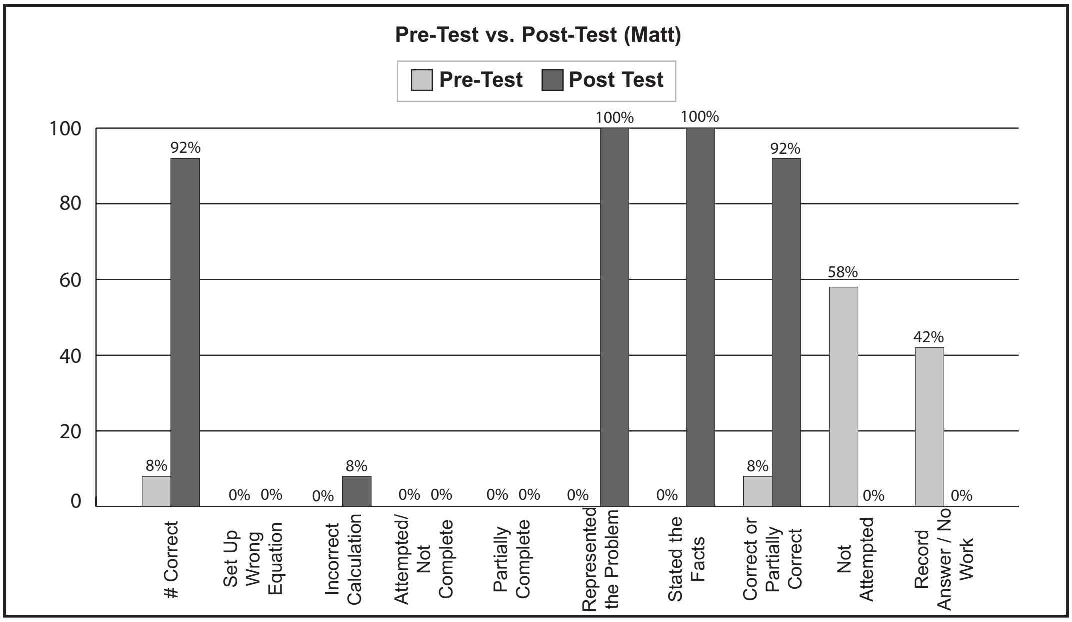


Figure 1

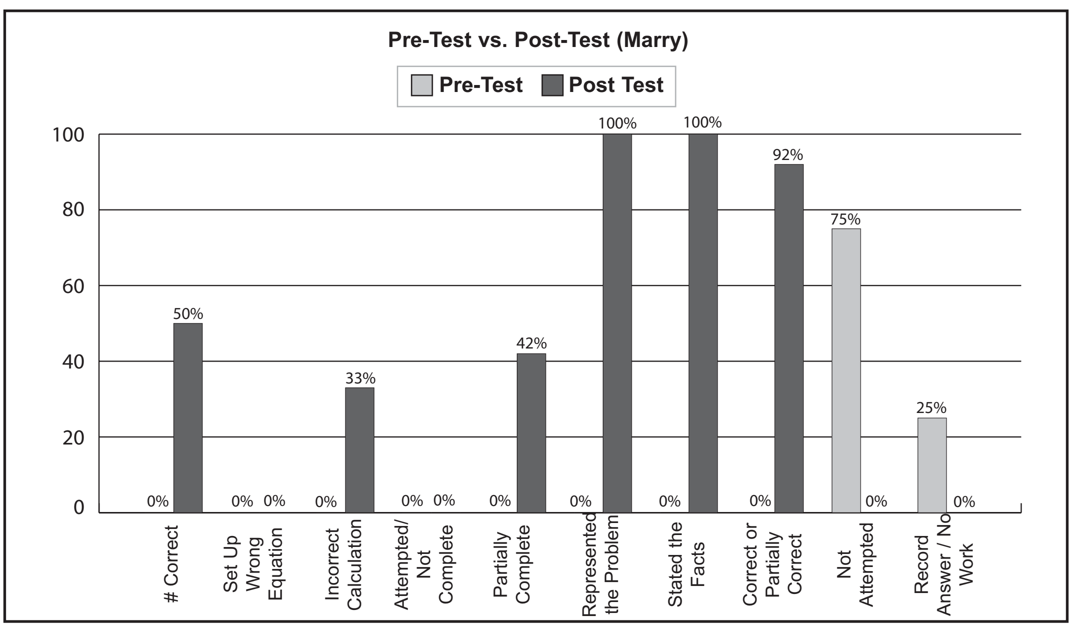


Figure 2

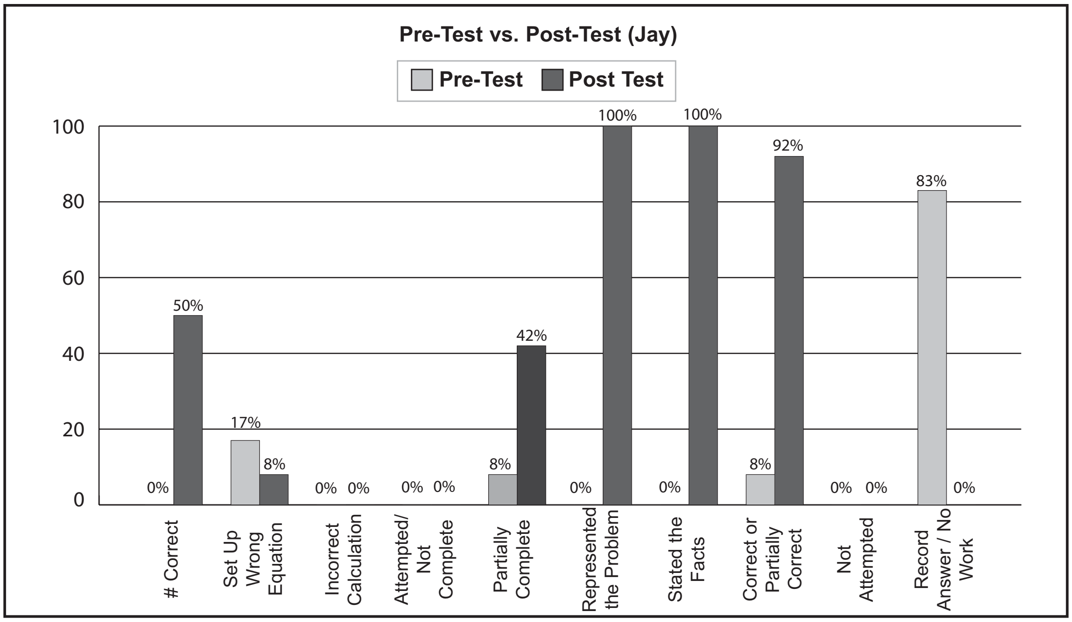


Figure 3

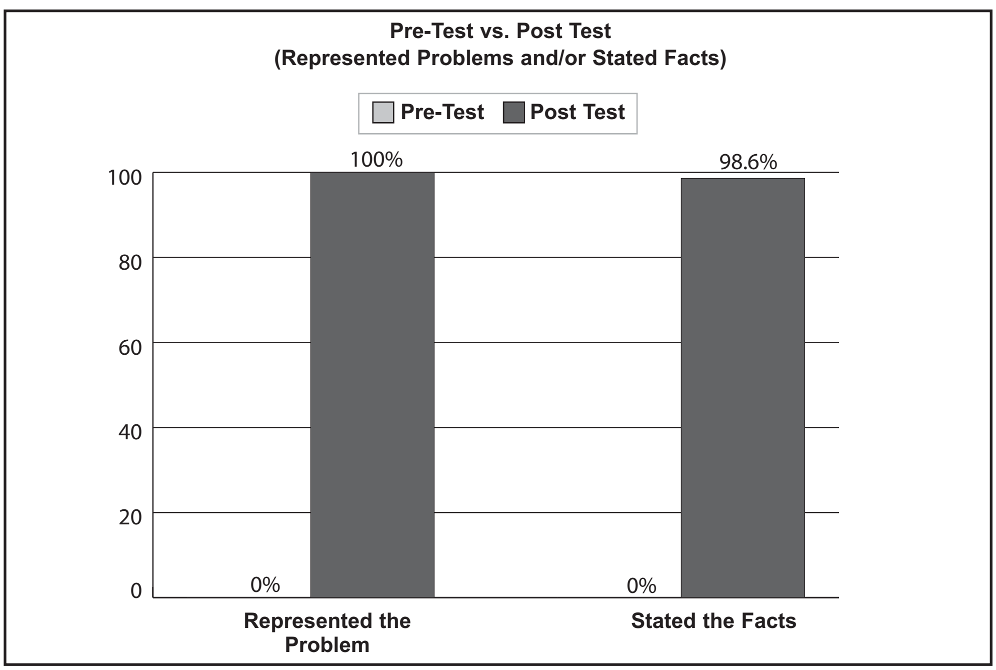


Figure 4

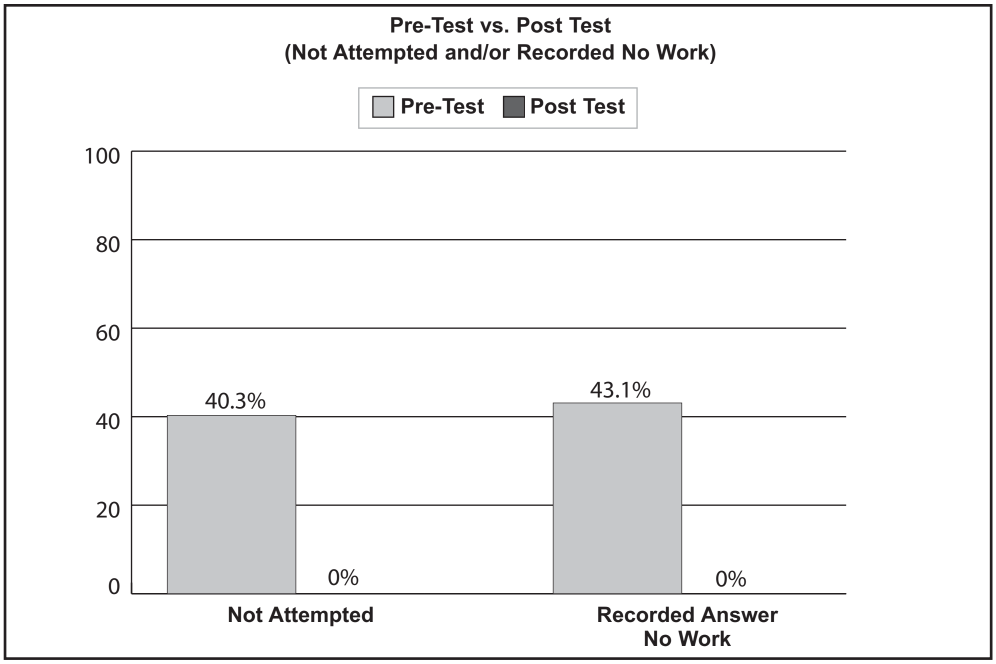


Figure 5

All Students' Data Represented as
Percentage on Pre-Test vs. Percentage on Post-Test
(6 Students at 12 Questions Each = 72 Questions)

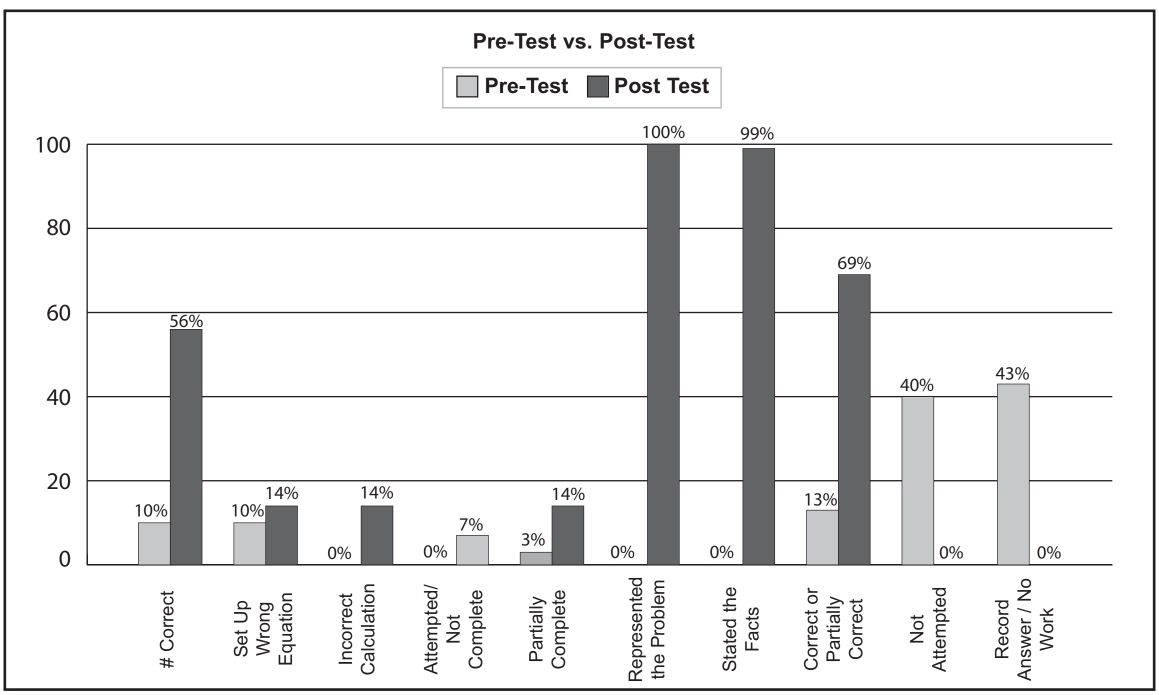


Figure 6

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